Friction Factors for Pipe .Flow

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The object of this páper is to furnish the engineer with a simple means of estimating the friction factors to be used in computing the loss of head in clean new pipes and in closed conduits runuing full with steady flow. The modern develop:ments in the application of theoretical hydrodynamics to the fluid-fr:ictjon problem are impressive and¡r scattered through an extensiva literatare. This paper is not intended as a critica! snrvey of this wide field. .For a concisa review, Professor Bakhineteff's (1)[2] sman book on the :mechanics of fluid flow is an excellent reference. Prandtl and Tietjeris (2} and Rouse (3) have also made notable contributions to the subject..The author does not clabn to offer anything particularly new or original, his abn merely being to embody the now accepted conclusions in convenient form for engineering use.

N the_ present pipe-.flow study, the friction factor, ·denoted by *fin*.the accompanying charts,.is the coefficient in the Darcy formula

$$h_i = f\frac{LV^2}{D\,2g}$$

in which h_i is the loss of head in friction, in Jeet of fluid column of the fluid flowing, L and D the lengtli and interna! diameter of t pe:-in-feet; V.the mean velocity of flow in feet per second; and g the acceleration of gravity in feet pei.: second per second (mean value taken as.32.16). The factor f is .a dimensionless quantity, and at ordinary velocities is a function of tw'o, and only two, other dimensionless quantities, the relative rougluiess of the surface, ϵ (e being a linear quantity in: feet representativa of the absoluta roughness), and the Reynolds number $R = VD/\nu$ (v being the- coefficient of kinematic viscosity of the fluid in. square feet per second).. Fig. 1 gives numerical values. *off* as a function of ϵ and **R:**

Ten years ago R. J. S.. Pigott (4) published a chart for the s e ,friction factor, usÍng the same co-ordinates as in 'Fig. 1 of this paper. His chart has proved to be, Ínost useful and practica! .a d has been reproduced in .a nn.mb r of texts (5). The Pigott chart was based upon án an ysis of some 10,000 experimenta from various sources (6), but did nat havé the benefit, in plotting or fairing the curves, of later developments in functionalforms of the curves.

In the same year Nikuiadse (7) published his experimenta on artificially roughened pipes. Based upon the tests of Nikuradse and others, von Kármán · (8) and Prandtl (9) developed their theoretical analyses of pipe flow and gave us suitable formulas with n.umerical constants for the case of perfectly smooth pipes or those in which the irregularities are small compared to the thickness of the laminar boundary !ayer, and for the case of rough pipes where the roughnelises protrude sufficiently t() break up the laminar !ayer, and the flow becoines completely turbulent.

The analysis did not, however, cover the entire field but left a gap, namely; the transition zone between smooth and rough pipes, the region of incompleta turbulence. Attempts to fill this gap by the use of.Nikuradse's resulta for artificial roughness produced by closely packed sand grains, were nat adequate, since the re-: sults were clearJy at variance fróm actual experience for ordinary surfaces encountered in practice. Nikuradse's curves showed a sharp drop followed by a. peculiar reverse curve,[3] not observed with commercial surfaces, and nowhere suggested by the Pigott chart based on many tests.

Recently Colebrook (11), in collaboration with C. M. White; developed a function which. gives a practica!· form. of traruiition curve to bridge the gap. This function agrees with the two extremes of roughness and gives values ih very satisfactory agreement with actual measureinents on most forros of commercial piping and usual pipe surfaces. Rouse (12)·has sho-wn that it is a. reasonable and practically adequate solution and has plotted a chart. based. upon it. In arder to .simplify the plotting, Rouse adopted co-ordin tes inconvenient for ordinary engineering use, since f is implicit in both co-ordinates, and **R** values are representad by curved co-ordinates, so that interpolation is ttouble-: sorne.

.The author has drawn up a new chart, Fig. 1, in the more conventional form used by Pigott, taking advantage of the functional relationships established in recent years. Curves af f versus **R** are plotted to logaritpmic scales far vario'us const11.nt values afrelahlve roughness ϵ; and to permit easy selection of an accompanying chart, Fig. 2, is given from whichi canpe read for any size.of pipe of a given type of surface.

In arder to find the frlction loss ·in a *pipe,* the procedure *is* as follows: Find the appropriate D^e from Fig. 2, then follow the corresponding line, thus identified, in Fig. 1, to the value of the Reynolds nutnber **R** corresponding to the velocity af flow. The factor f is thus faund, for use in the Darcy formula

$$h_i = f\frac{LV^2}{D\,2g}$$

In Fig. 2, the scales at the top and bottom give values of the diameter in both feet and inchés. Fig. 1 involv.es anly dimensionless quantities and is applicable in any system of units.

To facilitate the calculation of **R,** auxiliary scales are shown at the top of Fig. 1, giving values of the product (VD'') for two fluids, i.e., water and atmospheric air, at 60 F. (D'' is the inside diameter in inches.) As a further auxiliary,. Fig..3 *is* given, from which **R** can be quickly foui:td for water at ordinary temperaturas, for any size of pipe and mean velocity V. Dashed liD.es on this chart have been added giving values of .the discharge or quantity of fluid flowing, $Q = AV$, expressed in both ·cubic feet per second and in U. S. gallons per minute.

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• Numbers in parentheses refer to the Bibliography at the end of the papar.

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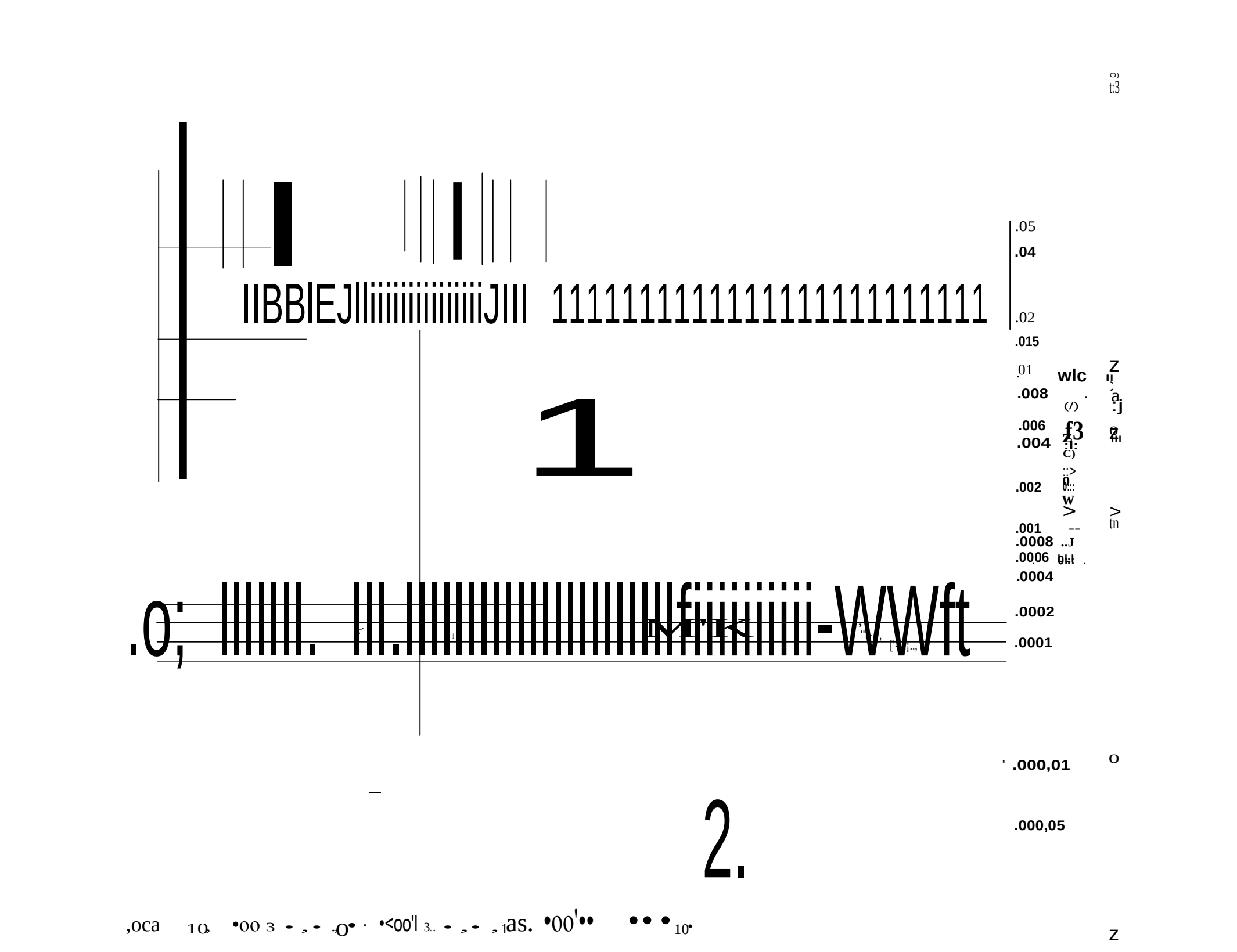
NoTE: Statements and opinions advanced in papera are to be understood as individual expressions of their authors and not those of the Society.

• Reuse, reference (3), p. 250; and Powell, referen,ce (10), p. 174.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- REYNOLDS NUMeER R,:; (V IN• > IN FT., V IN 5.)-_

: FIG. i -

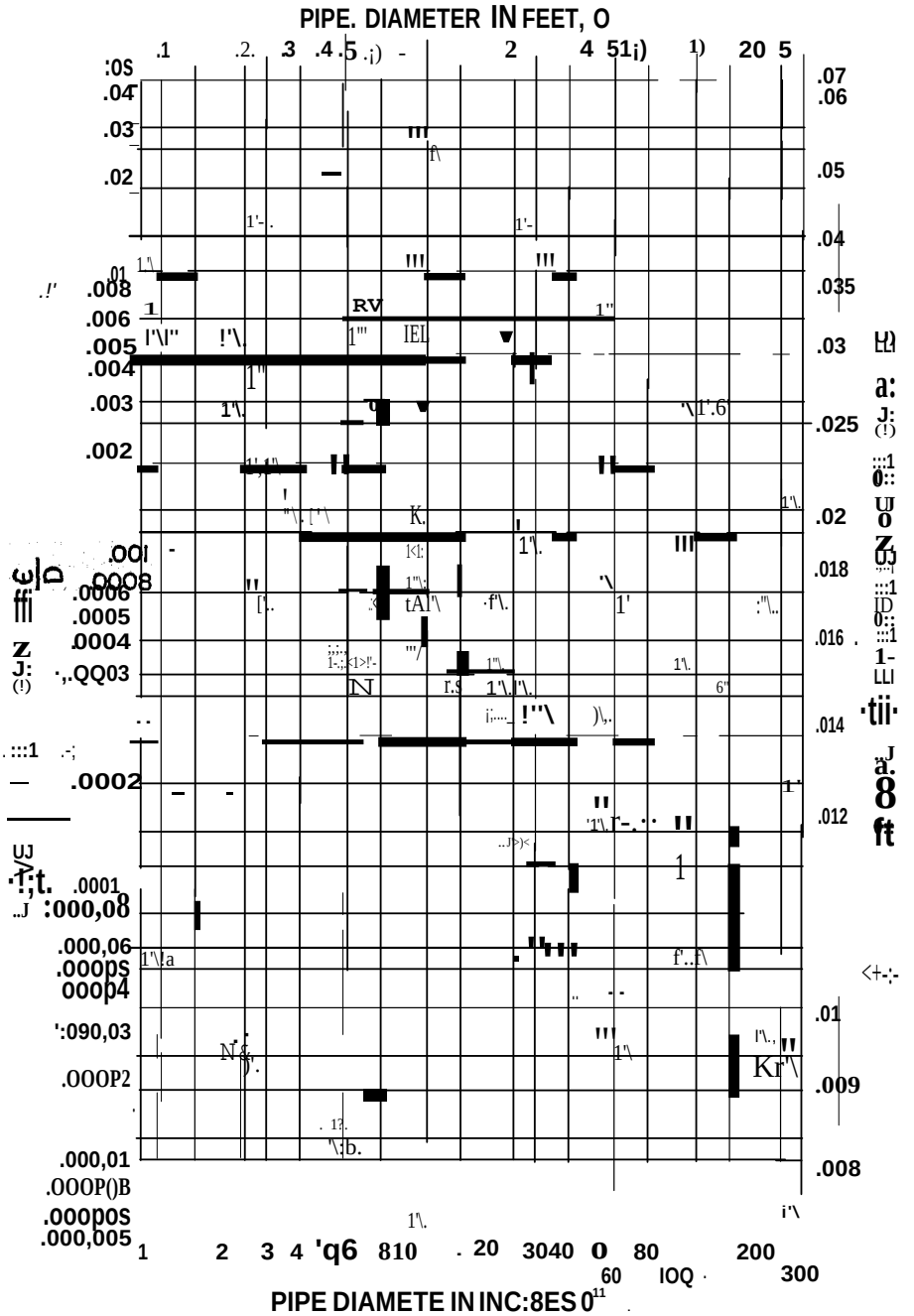


FIG. 2.

For other :fiuids, the ldnematic viscosity *v* may be found from Fig. 4, which with. Prof. R, L. Datigherty's kind permissiorÍ has been reproduced.[4] To enable **R** to be qirickly found for various :fiuids, Fig. ·4 includes an auxiliary diagram constructed by Dr. G. F. Wislicerius, which gives **R** for various values of the product *VD"* shown by the diagonallines. - For any value of *v* in the left-hand diagraÍn, by following a horizóntalline to the appropriate diagonal at the right, the correspo:ilding **R** may be read at the top of the auxiliary graph.

/Over a large part of Fig. . an approximate figure for **R** is suffi.cient, since *f* varíes only slowly with changes in **R;** and in the rough-pipe zone.j is independent of **R.** From the last coiisideration, it becomes ppssible to show, in the right-hand margin of Fig. 2, values of *f* for rough pipes and. complete turbulence.

• Reference (13) andreference (5).

If it is seen that the conditions of any problem clearly fall in th zone of conwlete turbulence abo've and to the right of the dá.shed line in Fig. 1, then Fig. 2 will give the valúe off directly withoút Jurther reference to t;he other qharts.

lLLusTRA.TION oF UsE oF CarnTs

Example 1: To estimate the lo;s of head in 200 ft of 6-in.' asphalted cast-iron pipe carrying water with a mean velocity of 6 fps: In Fig. 2, for 6 in. diam (bottom,scale), the diagonal for "asphalted cast iron" gives D = 0.0008 (left-hand margin). In Fig. 3, for 6 in. diam (left-hand margin), the diagonal for V = 6 fps gives $\mathbf{R}$ = 2.5 (10') (bottom scal!ll) {or, jnstead ofusing Fig. 3, compute VD'' = 6 X 6 = 36). In Fig. 1, Iooate from the right-

hand margin the curve for $\frac{D}{\epsilon} = 0.0008$ and follow this curve to a point above $R = 2.5$ (10^6) on the bottom scale (or below $VD'' = 36$ on the top scale). This point gives $f = 0.02$ (left-hand margin); then

$$h_f = f \frac{L}{D} \frac{V^2}{2g} = 0.02 \left(\frac{200}{0.5}\right) \frac{(6)^2}{64.3} = 4.5 \text{ ft friction loss}$$

Example 2: To estimate the loss of head per 100 ft in a 15-in. new cast-iron pipe, carrying water With a mean velocity of 20 *fps:* In Fig. 2, for 15 in. diam (bottom scale), the diagonal for "cast iron" gives $\frac{D}{\epsilon} = 0.0007$ (left-hand margin). In Fig. 3, for 15 in. diam (left-hand margin), the diagonal for $V = 20$ fps gives $R = 2$ (10^6) (or, instead of using Fig. 3, compute $VD'' = 20 \times 15 = 300$). In Fig. 1, the curve for $\frac{D}{\epsilon} = 0.0007$ (interpolating between 0.0006 and 0.0008, right-hand margin), at a point above $R = 2$ (106) (bottom scale) (or below $VD' = 300$, top scale) gives $f = 0.018$ (left-hand margin). In this case the point on Fig. 1 falls just on the boundary of the region of "complete turbulence; rough pipes." Here R or VD' need only be approximated sufficiently to see that the point falls in the complete turbulence region, and f can then be found directly from the right-hand margin in Fig. 2 without further reference to Fig. 1; then

$$h_f = f \frac{L}{D} \frac{V^2}{2g} = 0.018 \frac{(100)(20)^2}{(1.25)\ 64.3} = 8.95 \text{ or, say, 9 ft friction loss}$$

It must be recognized that any high degree of accuracy in determining f is not to be expected. With smooth tubing, it is true, good degrees of accuracy are obtainable; a probable variation within about ±5 per cent (14), and for commercial steel and wrought-iron piping, a variation within about ±10 per cent. But, in the transition and rough-pipe regions, we lack the primary and obvious essential, a technique for measuring the roughness of a pipe mechanically. Until such a technique is developed, we have to get along with descriptive terms to specify the roughness; and naturally this leaves much latitude. The lines in Fig. 2 might be more graphically represented by broad bands rather than single lines, but this is not practical due to overlapping.

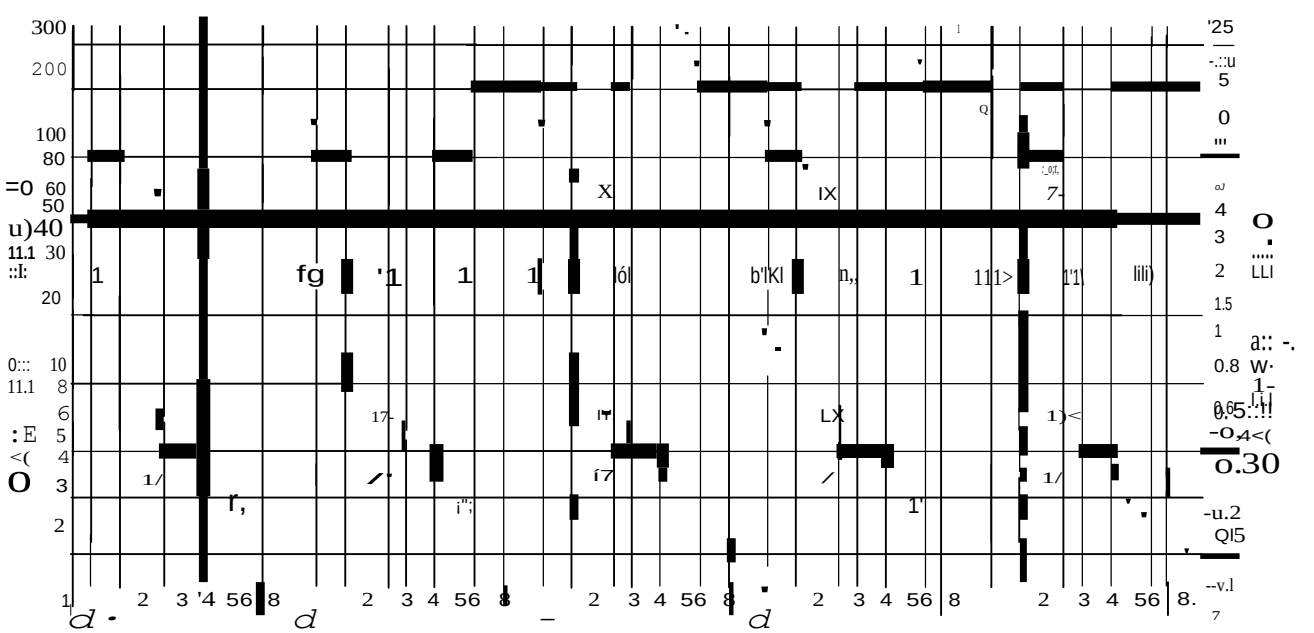
Even with this handicap, however, fairly reasonable estimates of friction loss can be made, and, fortunately, engineering problems rarely require more than this. It will be noted from the charts that a wide variation in estimating the roughness affects f to a much smaller degree. In the rough-pipe region, for the reasons just explained, the values of f cannot be relied upon within a range of the order of at least 10 per cent.

The charts apply only to new and clean piping, since the rapidity of deterioration with age, dependent upon the quality of the water or fluid and that of the pipe material, can only be guessed in most cases; and in addition to the variation in roughness there may be, in old piping, an appreciable reduction in effective diameter, making an estimate of performance speculative.

Although we have no accepted method of direct measurement of the roughness, in any case where we have a sample of pipe of the same surface texture available for test in the laboratory or in the field, then from a test of such a pipe in any size we can, by aid of the charts, find the absolute roughness corresponding to its performance. Thus we have a means for measuring the roughness hydraulically. The scale of the absolute roughness e used in plotting the charts is arbitrary, based upon the sand-grain diameters of Nikuradse's experimenta.

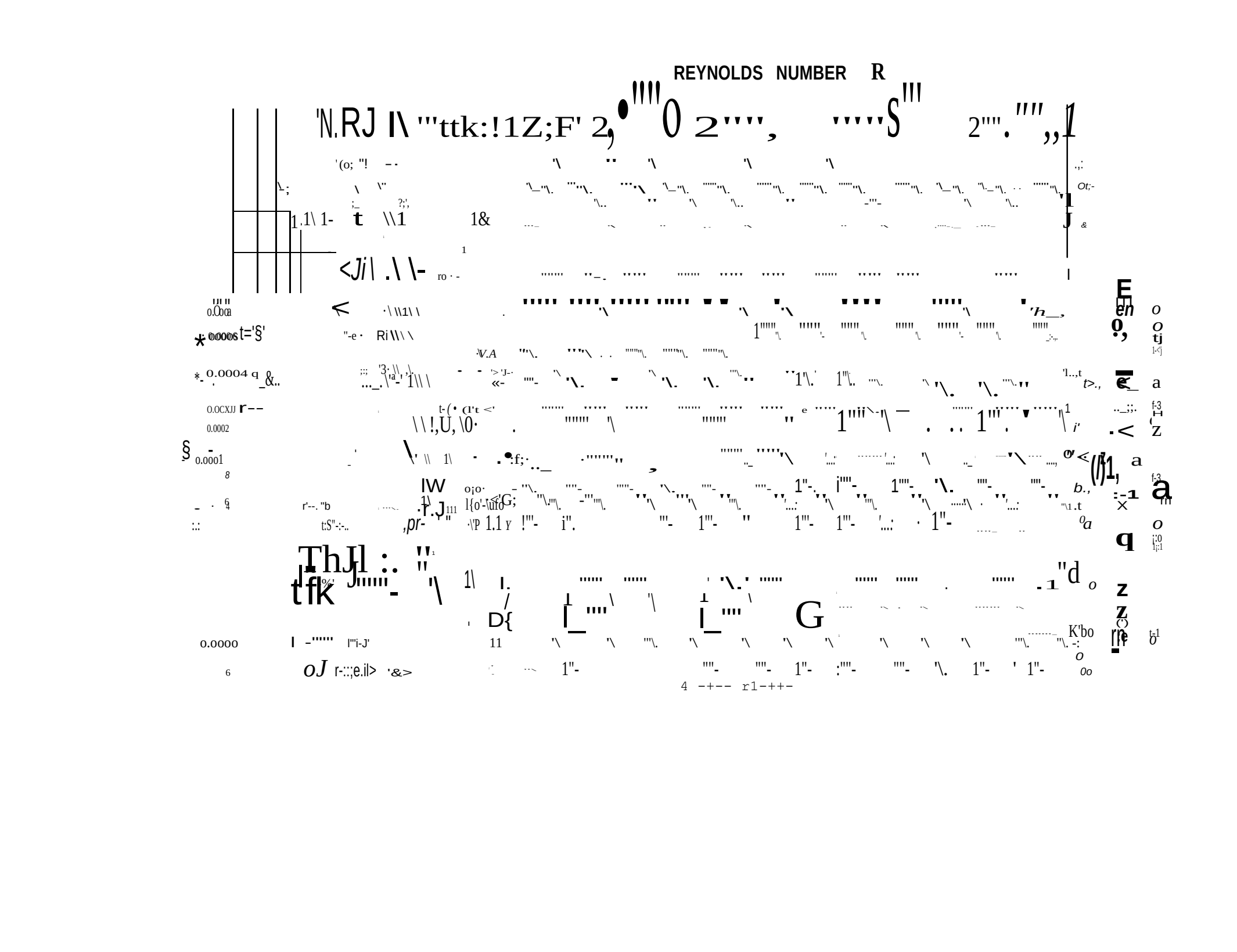
The field covered by Fig. 1 divides itself into four areas representing distinct flow characteristics. The first is the region of laminar flow, up to the critical Reynolds number of 2000. Here the flow is fully stabilized under the control of viscous forces which damp out turbulence, permitting a completely rational solution. The values of f are here given by a single curve, $f = R'$ independent of roughness, representing the Hagen-Poiseuille law.

Between Reynolds numbers of 2000 and 3000 or 4000, the conditions depend upon the initial turbulence due to such extraneous factors as sudden changes in section, obstructions, or a sharp-edged entrance corner prior to the reach of pipe considered;



REYNOLDS NUMBER $R = V.S.$ FOR WATER AT 60°F (V IN FT. PER SEC.; D IN FEET; $V = 1.216(10^{-5})$; $R = 68500 VD$)

FIG. 3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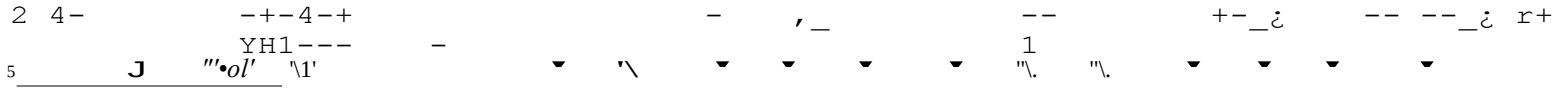

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JO w

VALUES OF {VD") (V IN::c.. X O•• IN INCHES)

Fra. 4

and the conditions are probably also 'a.ffected by pres ure waves . · initiating instability. This region has been called a critica! zon:e, and the indefiniteness of behavior in this region has been indicated by a hatched area without definite *f* lines. The minimum limit. for *f* values is the dotted continuation of the laminar-flow line, corresponcling to véry smooth and steady initial flow. When . there is distinct turbulence in -the entering fluid, the flow in the critica! zone is likely to be pulsatbg (2)rather than steady. The effects of strong initial turbulence may even extend into the laminar-flow zone, raising the *f* values somewhat, as far as to a Reynolds number of about 1200.

Above a Reynolds number of 3000 or 4000, conditions again become reasonably déterminate. Here we find two regions, namely, the transition zone and the rough-pipe zone. The transition zone extends upward from the line for perfectly smooth pipes, for.which the equation is

$$1/\sqrt{f} = 2\log \frac{Rv\sqrt{f}}{2.51}; \text{ or } 1/\sqrt{f} = 2\log R_{\sqrt{f}} - 0.8$$

(Kármáii, Prandtl, Nikuradse) to the dash d line indicating its upper limit, plotted from the relation·

$$v'J = \frac{1}{200}\,\eta$$

(following .. the corresponding line in Rouse's chart, reference 12). In the trari.sition zone the curves follow the Colebrook function

These clirv'es are. asymptotic at one end to the smooth pipe line B.Rd at tlie other to the horizontal lines of the rough-pipe zone. Actually, :the curves converge rapidly to these limits, merging with;the smooth pipeline at the left, and at the right, beyond the dashed line, becoming indistinguishahle from the constant *f* lines for rough'pipe.

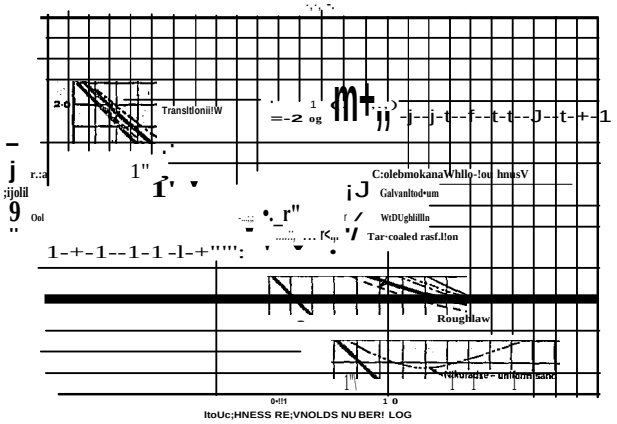
..: THE CoLEBR mr. FuNcTION

The basis of the Colebrook function may be briefly outlined. Von Kármán had'· shown that, for completeíy turbulent flow in rough pipes, the eipression $1/v1''' - 2.\log (D/2\bullet)$ is equal toS: · constant (1.74), or, as expressed by Colebrook, $2 \log \frac{3.7D}{e}$ is equal to·zero; ' In the transition regiOIJ. of incomplete turbulence van Kárlnán's expression is not equal to a i:lonstant but to'some function of the ratio of the size of the roughnesses tó the thickness of the laminar boundary lay r. Accordingly, Nikuradse had representad his experimental results on artificially rougliened pipes by plotting $1/\sqrt{f} - 2\log (D/2\bullet)$ .versus e/0, in which 0 is tii.e lainina r !ayer thickness. By .this ni.ethod of plotting, the results for all types of flow and degrees of roughness were shown to fall on a single curve. . Using logarithmic scales, the smooth-pipe curve becomes an inclinad straight line, and the rough-pip!) curves :ro,erge in a single horizontalline.

Colebrook (11), using equivalent co-ordinates, [6] plotted in his Fig. 1, here reproduced as Fig. 5, the results of many groups of tests on various types of commercial pipe surfaces. He found that each class of commércial pipe gave a curve of the same forro, and while these curves are quite different from Nikuradse's sand-grain results, they agree closely with each other and with a curve representing his transition function..

s'e/C! may be expressed in alternativa forma as proportionah to RVJ ṁ hich $r = D/2$, $k = \bullet$; orto $\frac{V \cdot k}{\nu}$ ṁ hi'n v* =.', Ed'

τ_0 being the shearing stress at the pipe wall, ρ the mass density of the fluid and μ *it.s* absolute or dynamíc viscosity.



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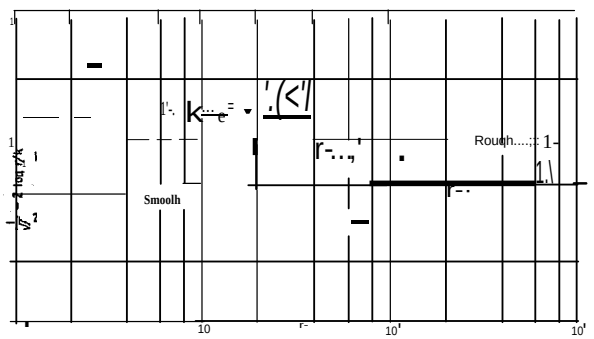

Fra. 6

Rouse '(12), also using equivalent co-ordinates, has plotted in: his Fig. 6, here reproduced as· Fig. 6, a large number of points each of which representa a series of tests on a given size of commercial. pipe, together with the Colebrook curVe. As he.points out, the deviation of the points from. the Colebr ok curve "is evidently not i'nuch greater. than the e:¡q¡erimental scatter·of the fudividual measurements in any one series."

·.When tlie tliickness of the laminar layer,. which decreases with increasing Reynolds number, becomes so small, compared to the surface irregularities, thab. the laminar flow is broken up inbo turbulence, the flow conditions pass over into tlie zone «;>f "rough pipes," with complete turbulence established practically throughout the flow; Viscous forces then become negligible ·ccnÍlpared to inértia forces, and *f* ceases tó be a function of the Reynolds number and depénds only upon the relativ roughnessr giving.horizontallines of constant *fin* the chart. These !ines agree with the von Kármán rough-pipe formula

$$1/\sqrt{f} \quad 2\log\left(\frac{r}{\epsilon}\right), \text{ or } 1/\sqrt{f} - 1.74 - 2\log\left(\frac{2r}{\epsilon}\right)$$

Since *!* depends upon the relative roughness, the ratio of the absolute roughness to the pipe diameter; even a fairly rough surface in a very large pipe gives a small relative roughness. Thus Colebrook plots the results obtained on the penstocks of the Ontario Power Compan;v.; where metal forros and specially laid concrete produced a very· smooth example of concrete surface. This in combination with the large diameter gave a, relativa roughness comparable to drawn brass tubing, with/ values falling practically on the "smooth pipe" line of Fig. 1. Such specially fabricated welded-steel pipe lines as those of the Colorado aqueduct system would probably give values along the same curve.

On the other hand, at very high velocities in drawn tubing of

small diameter, eventhe smaJl absolute roughness is suffi.cient to brea}!: up the laminar boundary !ayer, and the tubing becomes in effect a "rough pipe." Very few experimenta have carried the velocities and Reynolds numbers high enough to permit a clase estimate of ε for drawn brass, copper, or similar tubing; but by applying the Colebrook function to the available data_ (14, 15), for the smocithest surfaces reported upon, ε was .estimated as of .the arder of 0.000005; and a line corresponding to this value has been drawn in Fig. 2, serving as a minimum limit for surfaces likely to b encountered in practice.

PIPE FRICTION FACTORS APPLIED TO 0PEN-ÜHANNEL FLOW

Pipe friction factors have sometimes been applied to open-channel fiow; and more commonly the friction losses in large pipes and other closedf conduits have been computed from open-channel formulas. The Chezy formula for open channels is $V = e\text{-}•l/mS$ in whil'h Vis the mean velocity; m the hydraulic mean depth or "hydraulic radius," the sectionhl area divided by the wetted perimeter; S the slop , the loss of head divided by'Iength of channel, and e a coefficient. . The Chezy formula is equivalent to the Darcy formula for pipes, the Cliezy coefficient e being convertible futo f by the relation $f = -'$ It should be considerad, however, that the Chezy coefficients have'been derived principally from observations on relatively wide and ·shaJlow channels of large area and rough bottoms, far from circular in shape, and that they involv a free water surface not present in closed conduits, so that; even when the·fiow is unifor'm, the problem is highly complex. Consequently,.such formulas as Manuing's are recommended for open channels in preference to the use of .values of e derived from the pipe friction factors.

Open channels dealt with in engineering practice -a:re usually rough;surfaced and of large cross section, corresponding to large Reynolds numbers and falling in the zon'e of complete turbulence, so that the friction factors are practically indepimdent of Reynolds number. · The presence of a free surface, however, with surf ce waves or disturbances, introduces a consideration not involved· in closed-conduit fiow. It is therefore the author's view that while, for open channels, we can drop the Reynolds number as an index of performance, we should replace it by a new criterion, the Fraude number relating the velocity head and ·depth, whieh can be expressed as $\frac{V2}{2gm}$ or $\frac{V}{\sqrt{gm}}$; or more strictly $\frac{V}{\sqrt{gm}}$; in which o denotes t e average dep h or .sectional area divided by the surface breadth; the latter forro representing the ratio of mean velocity to the gravitational critica! velocity or velocity of propagation of surface waves.

This proposed criterion defines whether the fiow falls in the "tranquil," "shooting," or critica! state. The neglect of this factor may at least partially accourit for inconsistencias ·between various open-channel formulas; and between open-channel and pipe-Jriction formulas, and casts particular doubt on accepted formulas for open-channel friction in the critica! or shooting-fiow regions. These considerations suggest the plotting of open-channel friction factors as a function of the relative roughness and the Fraude number, in similar manner to the plotting *off* as a function of the relative roughness and the Reynolds number for closed conduits.

For the foregoing reasons, Fig. 1 is not reco=ended with much con:fidence for general application to open channels, for which 'a formula such as Manning's better representa the available information. The charts can however be•applied, at least as an approxiiDation, to noncircular closed conduits of not too eccentric a forro or not too different from a circular.section, by using ab equivalent diameter

$$D = 4m = 4\left(\frac{\text{Sectional area}}{\text{Length of perimeter}}\right)$$

Since civil engineers usually classify surface roughness by the Kutter .and M¡¡,nning roughness factor n, it would be helpful in selecting a value of e for such variable surfaces as concrete, if *we* could correlate e and n. P. Panagos[6] has applied the Colebrook function to the test data collected by Scobey (16) and finds the following values of e correspondiríg to the Kutter n ratings given by Scobey, which may be at least tentatively utilized:

Kuttern	0.0105	0.01.1	0.012	0.013	0.014	0.015	0.016
Absoluta roughnesse	0.00015	0.0005	0.002	0.005	0.011	0.02	0.03

Accordingly, on the basis of Scobey's data the lines given in Fig. 2 for·concrete may be somewhat more definitely described. as follows:

n		
·0.001	O. 0115	{Highest practica! grade of concrete. Buriarie and joint
0.003	O. 0125	smooth
0.01	O. 014	Concrete surface with alight form ma.rks, fairly smooth. joints or rqughly troweled
0.03	O. 016	Prominent form ma.rks or deposita of atonas on bottom

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Discussio"n

R. L. DAGHERTY.[7] The writer has nothing but commenda-tion for this excellent paper. The author has presentad the latest theory combinad with the available experimental data in a manner which makes it more convenient for use than has been the case heretofore. His evaluation of relativa. roughness for di:fferent types and sizes of pipes is· a step forward.

While this paper deals primarily with pipe friction it is interesting to note the suggestions made concerning the treatment of the flow in open channels. The latter has not been given the attention from the standpoint of rational analysis that has been devoted in the past to pipes. Itis to be hoped that developments in this field may be made along the lines suggested by the author.

The author calls attention to the well-known fact that in the transition zone the Nikliradse curves for his artificial sand-grain roughn ss are quite di:fferent from those obtained with commercial pipes. The writer would like to know if the author lías any explanation to offer for this marked difference.

C. W. HUBBARD.[8] This paper is of interest to.engineers who must estímate fluid-friction loss closely for certain types of problema. Ordinarily the Manning type of formula is preferred, since the roughness value may be determinad from the type of surface of the wall as contrastad to the Darcy formula where the roughness coefficient varies with the size of pipe and is di:fficult to estiinate. The author's Fig. 2 allows a quantitative wall roughness estimated from the type of wall to be used.

During some recent tests made to select a protective paint for steel which would also have a low friction loss, it was found that severa! coatings, particularly those consisting of certain bitumastic constituents which required them to be applied thickly to the wall, gave low flow-resistance values. The tests, made in 3-in. pipes, which were ·split longitudinally to allow proper application oí the coating, showed roughness values of the arder of those obtained with drawn-brass tubing. However, the appearance of the coating was not as smooth as drawn tubing. The writer had previously experienced this effect with similar coatings. There seems to be little published material on the friction loss produced by various protective paints .and coatings on pipe walls, particularly on small pipes when the flow is likely to occur in the transition range where the friction loss is dependent upon Reynolds number. Apparently the roughness of such surfaces is of the waVY type which cannot be evaluated on the same basis as the same magnitude of roughness which is of the granular type.

A. T. !PPEN.[9] The author has ably satisfied the object of his paper stated in the beginning with an extremely timely and practica! summary of the latest information available on pipe friction. Academic research in this field over the last 30 years conducted on a fundamental basis has finally yielded a satisfactory explanation of the nature of the laws of pipe friction and has cleared up the concepts of energy dissipation in conduits and channels.

The evidence for the adoption oí the methods for. determining the pipe friction factor as presentad by Colebrook is rather astanishing. Some experiences in this connection may be contribut d here. The writer has computad two comprehensive sets of data .on ,pipe friction, one by John R. Freeman and ari-other by L. H. Kessler. The former completad his experiments during the years 1889 to 1893 and hi data were ·published by this Society in a special volume (15)[10] l.n 1941. .The second set of data was obtained from pipe-friction experimenta at the Wisconsin Experimental Station, the results of which were published in '1935. Both experimenters performed tests on 1/4 in- to 8 in- diam wrought-iron pipes in new condition covering the maximum range in Reynolds numbers possible under their experimental conditions. After plotting these ·results ev ry one oí their. runs shows essentially the f versus **R** curve indicated by Colebrook and e values calculated for all the various sizes come out very clase to the average value stated for wrought-iron pipe in the present paper. It must be remembered that Kessler's data were obtained 40 years after those of Freeman and that it can hardly be assumed that manufacturing processes remained identical during that perlad.

Another fact of importance to the practica! engineer from this analysis of Freeman's and Kessler's data is worth mentioning. Rolise and Moody in their f versus **R** curves terminate the transition range from smooth- to rough-pipe flow along a line corresponding to a ratio of absoluta :roughness e to the laminar boundary-layer thickness 0 of 6.08. Kessler's and Freeman's data do not give a single value that liigh in all their runs; their highest values obtained were about $\frac{e}{\delta} = 2.5$. Under practica] co dition8 of use therefore the flow of water in pipes occurs well in the transition range from smooth- to rough-pipe flow.

This fact easily explains why a final solution of the pipe-friction problem was possible only after the concepts of "smooth-pipe" and "rough-pipe" flow had been established separately. While Nikuradse's results on uniformly sand-coated pipe were helpful in this respect, they also resultad in móre complicated transition curves than are obtained from tests with the statistical roughness patterns encountered *o:b.* most commercial pipe surfaces. The Colebrook universal function seems to fit the better in this transition range; the more the roughness irregularities are statistically distributed as far as size and shape are concerned and vice'Versa, the more regular the size and pattern of the irregularities the closer Nikuradse's transition curves are approached, where finally the critica! velocity for all roughness bodies is the same in the ideal case of completely uniform size.

The fainiliar functions for the pipe_friction factor f may be written in the following forro

$$\frac{1}{\sqrt{f}} = 1.74 - 2\log\frac{R}{\epsilon} \quad \ldots\ldots\ldots [1]$$

for smooth pipe flow

$$\frac{1}{\sqrt{f}} = 1.74 - 2\log\frac{e}{\ldots} \quad \ldots\ldots [2]$$

for rough-pipe :fl.ow

$$\frac{1}{R\sqrt{f}} = 0.282\frac{r}{\ldots} \quad \ldots\ldots [3]$$

for laminar boundary-layer thickness.

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[10] Numbers in parentheses throughout the discussion refer to the Bibliography at the end of the author's papar.

According to Oolebrook, Equ tions, [1] and [2] are combined into the following universal function

$$\frac{1}{\sqrt{f}} = 1.74 - 2\log\left(\frac{18.6}{R\sqrt{f}} + \frac{e}{r}\right) \quad \ldots\ldots [4]$$

This function reverts to either Equation [1] or Equation [2], if either the influence of the relativ:e roughness disappears or when the viscous in:fluence becomes insignificant. By use of -Equation [3], the·Oolebrook function may be written in the alternativa form

$$\frac{V}{V_i} = f\,1.74 - 2\log\left(1 + \frac{0.282}{\frac{r}{e}}\right) \quad \ldots\ldots [5]$$

This equation clearly brings out the dependence of the pipe friction phenomena upon the thickness of the laminar boundary layer, i.e., on the viscosity of the :fluid. It will be found in practica! calculations that this influence is very seldom absent, The prop sed ultimate value of $\frac{RV\sqrt{f}}{\frac{r}{e}} = 400$ is equivalent to a value of of 6.08.

It is evident that aging of pipes under varying conditions of use will result i.p. new values of absoluta roughness which at present are not easily'predicted. From experimenta on galvanizad steel pipe of 4 fu. diam at the Hydraulic Laboratory of Lehigh University, an initial average valile of e = 0.00045 ft was obtained. This value of e was doubled within 3 years as a result of the change in surface conditions with aging under .moderate conditions of use. It must be remembered here that this change in e representa only about a 20 per cent increase in the Darcy-Weisbach factor /, since the e value is a much more sensitiva indicator of pipe roughness than the factor f.

In conclusion, it_ may be hoped that this paper will bring the general adoption of this relatively easy and reliable method of determining pipe friction and thereby establish a standárd procedure in practice which is based upon sound analytical and experimental evidence.

W. S. PARDOE.[11] In the following tests on pipes of various diameters and materials, the exponerit *n* in the exponential formula·

$$V = Kd^m S^n$$

varied from 0.535 to 0.546, thus checking Williams' and Hazen's formula

$$V = 1.318\,C\,r^{0.63}s^{0.54}$$

very closely. The maximum value of R was about 1,250,000 for 8-in. Neoprene dock-loading hose (very smooth) which is much below the "complete turbulence zone." The tests mcluded:

6-in. Italite cement-asbestos pipe (predecessor of Transite)
4-in. Ruberoid cement-asbestos. extruded pipe
4-in. fiber conduit
6-in. and 8-in. Neoprene dock-loading hose íor E. I. du Pont de Nemoms
2-in. to 12-in. stee1 pipe
8-in. rubber dock-loading hose with 1-in. X 1/8-in. helical metal band oninside

In no case except the last did the e:ll.--ponent *n* show even a tendency. of decreasing, let alone approaching a value of 0;5 or complete tmbulence. This must be due to the smoothness of the materials and the low value of Reynolds number. In the last case, the values *off* did show a tendenciy to become constant, the value of e/*D* being quite large.

The writer has not conducted a suffi.cient number of tests on pipes and is far from a pundit on this subject. At sorne time in the future, he will attempt to work into the "complete turbulence zone," if such there is, even if he must use a bit of 4-in. turberculated cast-iron pipe.

Mr. Pigott in his discussion has mentioned my insistence on the fact that the coefficients of Venturi meters become constant. This coe:fficient may be approximated by the formula

$$e = K\left(\frac{1}{d^4} + 1\right)$$

in which is the diameter ratio rh/d, and K is the coe:fficient of loss in

$$h = K\frac{V_2^2}{2g}$$

The value of K on the fiat part of tests of 85 cast-iron Venturi meters approximates

$$K = \frac{0.0435}{(d_2")^{0.25}}$$

As the absoluta roughness is constant, the proportional roughness varies inversely as the ·diameter or the coe:fficient increases with the diameter. The tests ran to quite high values of Reynolds number in terms of $V^2 rhP$, thu fu.dicating there is such A, thing as complete turbulence. Solving.the foregoing expression

$$K = \frac{1}{e^4}\left(1 - \frac{4}{2}\right)$$

Hence a constant value of *e* gives a constant *K*, or n_i varies as V^2.

This is of course arguing from the writer's experience with Ventilri meters to make up for his lack of adequate experimental knowledge of the subject under discussion.

Professor Moody says *f* is a function of "two and only two', dimensionless quantities *e/D* and $VD\rho$. The writer has foundin his work on Venturi meters a variation of qver ½ per cent,due to the effect of the ambient temperature.[12]

As a variati n of 1/2 per cent in *e* requires a variation of 25 per cent in *k* it seems to the writer the e:ffect of a di:fference of temperatura of 20 deg F *onf* at low value of R might be considerable. This e:ffect is brought about by a change in the boundary shear; thus

$$\tau_o = \frac{f\rho V^2}{g} = \mu\frac{dv}{dY}$$

If *Q* is kept constant *dvjdy* will also be cónstant, and *p.* corresponda to the temperatura of the inside wall of the pipe, which willlie between the ambient temperatura and that of the water. It will decrease as the velocity increases as a. result of the heat being conducted away more rapidly. This the writer will check in future experimenta; it may throw sorne light on the ilpper limit of the critica! or unstable zone. The e:ffect is a function of Reynolds and Pra:ndtl's or Nusselt's numbers, and the writer is not certain "what the price :of cheese in Denmark does to e:ffect *f*."

[11] Department of Civil Engineering, University of Pennsylvania, Philadelphia, Pa.

[12] "Effect of High Temperatmes and Pressmes on Cast-Steel Ventmi Tubes," by W. S. Pardoe, Trans. A.S.M.E., vol. 61, 1939, p. 247.

Professor Moody is to be congratulated on producing a very usable plot of friction factors which in due time may replace the Pigott and Kemler curves which have. to date been extensively quoted and used by engineers. Thus do we progresa.

R. J. S. PrGoTT.[13] This study of friction factor in pipes is particularly interesting to the writer, as it is a valuable further rationalization of a situation which has been unsatisfactorily empírica!. ·

At the time the writer's own correlation (4) was presi.mted (1933) there was almost complete lack of uniformity between various formuiations m general use, wandering all the way from Kutter, Hazen, and Williams tables, to Aisenstein's averaged values.

There was great need to prepare a formulation that would work satisfactorily for all kinds of conduit, from brass tubing to brick ducts, and for all fluida.

Dr. Kemler, then on the writer's staff, did the laborious part of the job, in correlating the resulta of all the experimenta published up to that time, culling all those with incomplete data (6). The writer summarized this work, in form for direct application generally, introducing the roughness e:ffect by rather strong-arm empirics, but at any rate the resulting chart worked well and has been growing in use.

.The great value of the author's study is that it puts the roughness effect, at last, on ·a much better justi:fied basis. For example, Huckingham (Fig.l, reference 5) drewthe lines for di:fferent sizes of steel pipe curved as they approached the viscous region, · the same as tb.e author now shows therri; Stodola (Fig. 1, ref. 5) •shows them straight and in'terceptiiig. , , . .

The later material used by the autb,or shows that they are curved. Another important point settled by the author is that the lines for all roughnesses iinally reach a constant value. The point at which this condition obtains is plainly shown to be a function of relative roughness, and so solves a difficulty Dr. Kemler and the writer had, in correlating sorne of the test material. Sorne of the experimental resulta showed rather ffat coefl:icients that were unexpected in regions oí moderate roughness. But this constant value of f is confirmed by Professor Pardoe's ·findings on Venturi discharge coefficients. He has been pointing out for years that the coefficient reaches a constant value at sorne Reynolds number that increases with size. Siri.ce most Venturis above rather small sizes are made with cast approach eones and the loases are substantially represented by pipe friction, this situation corresponda to fiat final value of f at complete turbulence, and a decrease of the value *off* with decrease of roughness.

Sorne engineers may be interested in the fiow of queer materials like. grease13, muds, cement slurries, etc., that have thixotropic properties (quoted from the rheologists), that is, they have plasticity mixed in with viscosity. All these materials have apparent viscosities ·which decrease with increase of shear rate, but, when they finally reach turbulent fiow, behave like true liquida of rather low viscosity. Such activities as oil-well drilling, cement-gun and grouting operations, automotive greasing equipment, an:d ball bearings involve such materials. In food industries, omi gets tomato ketchup and pea soup; glue and soap solu tio:D.s, paint and varnish operations, and various queer mate rials in the 'rayon and plastics industriés. For those interested, a paper[14] by the writer presenta more or less a rational solution that has been quite satisfactorily supported by tests.

In Fig. 1, the author has drawn his dotted line of complete turbulence somewhat in advance of the Reynolds number values at which the friction factor becomes a constant quantity. The writer finds that the expression

$$\frac{3500}{\frac{\varepsilon}{D}} = R_i$$

representa, as closely as can be determined·from the small-scale diagram, the point at which the friction factor becomes absolutely constant. It is curious and probably only accidental 'that the value 3500 corresponda about to the upper limit of the critica! zone.

HTINTEJR RousEJ.[15] Important resulta of laboratory research frequently do not reach the hands of practicing engineers untÜ many years ·after the original papers have been published. ·.A salient case in point is the discovery by Blasius in 1913 of the dependence of the Darcy-Weisbach resistance coeflicient f upon the Reynolds number R, which did not come into general engineering use until perhaps a decade ago. It often happens, however, that once engineers have accepted a new idea they are loath to modify it in any way. The paper under discussion is a very commendable endeavor to make recent experimental finciings immediately useful to the engineer, but the writer feels that it still caters to a regrettabíe degree to the engineer's innate conservatism.

If the writer's belief is correct; this paper is intended to fulfill the same purpose as that which prompted the writer to presenta somewhat similar paper (12) and resistance chart at the SecOnd Hydraullcs Conference in 1942. The author claims that·in this chart, which is reproduced herewith in slightly modi:fied farm, 1s the writer adopted co-ordinates inconvenient for ordinary engineering use. Such criticism resulted from the writer's deliberate advancement beyond the now familiar Blasius f-versus-R notation in the belief that both greater convenience and greater signi:ficance would be attained thereby. Since these two papers of identical purpose thus di:ffer in their basic method of approach, !J.. criticism of the one point of view must necessarily involve a de-.fense of thé other.

Although Blasius' original dimensional analysis of the variables involved led to his adoption of the form *VD/vas* the most signi:ficant grouping of terma upon which f should depend, it must be realized that the following three di:fferent combinations of the same variables are all equally valid for the basic case of a smooth pipe;[17]

$$f = \frac{2gh_fD}{LV^2} = \varepsilon p_i\left(\frac{VD}{\nu}\right) = \varepsilon p_i(R)$$

$$f = \frac{2gh_fD}{LV^2} = \varepsilon p_2\left(\frac{(2gh_f)^{\frac{1}{2}}D}{L\nu}\right) = \varepsilon p_2'(R\sqrt{f})$$

$$f = \frac{2gh_fD}{LV^2} = \varepsilon p_3\left(\frac{(2gh_f)^3D^3}{L\nu^2}\right) = \varepsilon p_3'(Rf^{\frac{3}{2}})$$

The combination now most familiar to the engineer, of course, is the first, although it has long since been proved that it Will.yíeld a linear plot on logarithmic paper for only the laminar zone. The second, on the other hand, is the basis of the Kármán-Prandtl analysis of the turbulent zone, the general furictional relatiOnship simply·being written in. the specific form

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'' "Mud Flow in Drilling," by R. J. S. Pigott, Drilling and Production: Practic. A.P.I., 1941, pp. 91-103.

1· Director, Iowa Institute of Hydraulic Research, University of Iowa, Iowa City, Iowa.

1· "Elementary Mechanics of Fluida," by Hunter Rouse; John Wiley & Sons, Inc., New York,' N. Y. (in Press).

17 "Solving Pipe Flow Problem With Dimensionless Numbers," by A. A. Kalinsk:i, *Engineering News-Record,* vol. 123, 1939, p. 23..

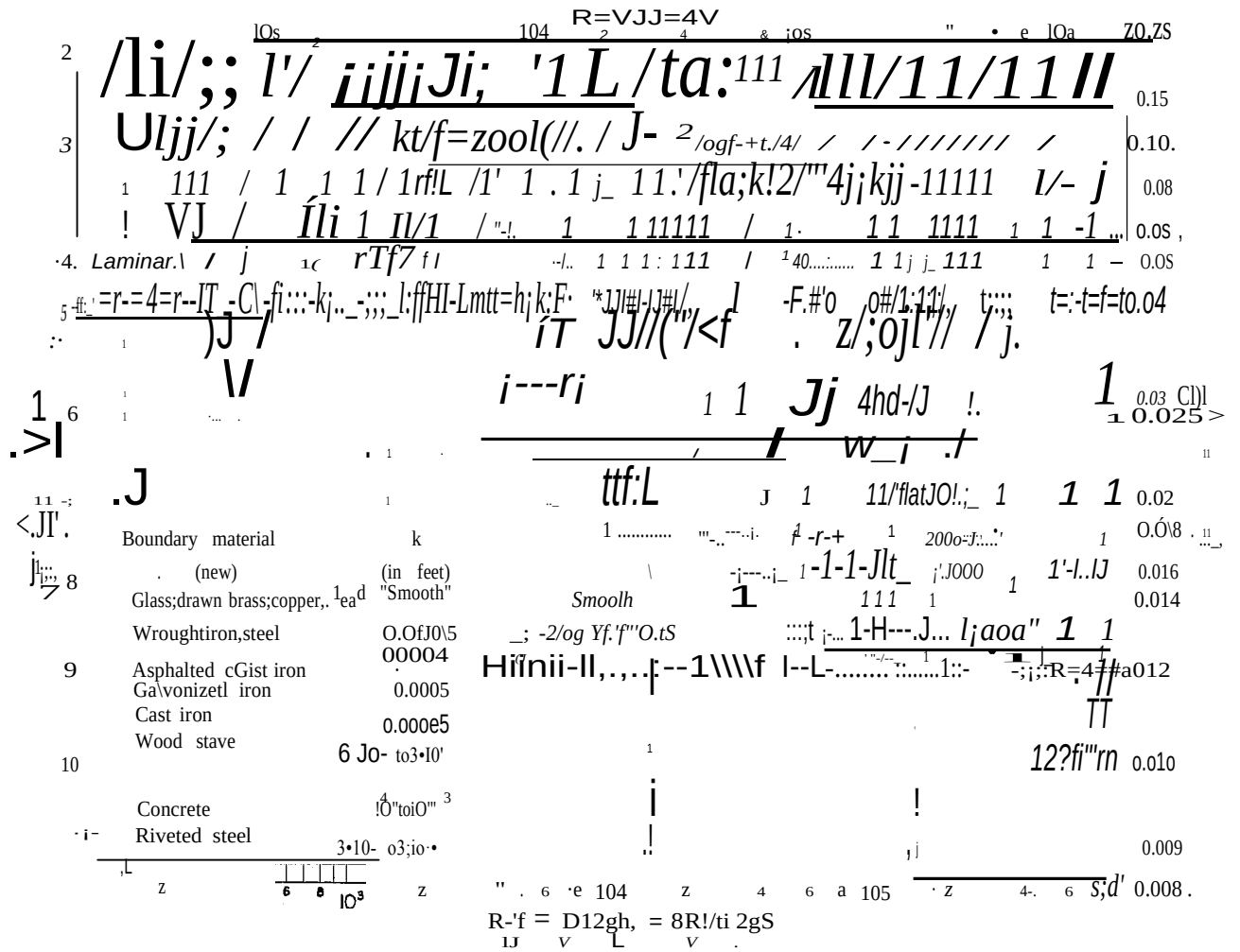

FIG. 7

$$1/VJ = A + B \log (Ry'j)$$

Despite the author's indication to the contrary, f is not inextricably embodied in the second term of this relationship, n.s may be seen from the identity $Rv'J = V'(2gh_iD^3/Lzi^2)$. If the Kármán-Prandtl parameters are chosen as the basis of a .semilogarithrcic chart, as in the accompanying figure, not only' will the smooth-pipe relationship plot as a straight Iine, but an transition curves from the smooth to the rough relationship will be geometricaliy similar. It would appear to the writer -that this combines ease in interpolation (and hence convenience) with greater significance than the Blasius plot will perrnit. This, therefore, is .one of the writer's two reasons for continuing to recommend the .newer type of chart in preference to that of the n.uthor.

The writer's second reason will be evident after further inspection oí the foregoing íunctional relationships. The :first relationship will be directly useful in graph forro only if the velocity or rate of fl.ow is known; otherwise the desired coefficient may be evaluated írom the graph only through the inconvenient process of trial and e;ror. Ií the velocity or rate of fl.ow is not known, on the other hand, a graph of the second functional relationship will make the desired coefficient imniediately available. In order to provide a single chart which would satisfy both sets oí requirements, the writer supplied ordinate scales oí both f and $1\sqrt{i}$ (the latter being proportional to th¡¡ Chezy C) and abscissa scales ofboth $R = VD/v$ and $RVJ = V2gh,/L\ n'!:v$. Since the parameters $llv'f$ and log .(RVJ) were seiected by the writer for the primary ordinate and ·abscissa scales,· the alternative abscissa parameter log R is necessarily represent!!Jd by .curved lines over a portion of the writer's chart. Had log f and log R been chosen as the primary parameters, log (R $\sqrt{f}$) would still have required sloping lines in the grid; such choice therefore involves no particular advantage over the writer's but rather defeats the writer's purpose owing to the accompanying distortion of the entire system of transition curves. The author's graph, of course contains no secondary grid system simply because it permits cfuect solution íor only one oí the severa! variables.

Brleí mention rnight .be made of the third possible combination of variables which is evidently applicable to problema in which the diamet r is the unknown quantity. So long as the pipe is smooth, such a plot will be of use, but the adoption of a similar function for the case of rough surfaces will still require a trial-and-error solution unless the graph is made hopelessly complex, owing to the .fa t that for á given boundary material the pipe diameter must be known. befare the relative roughness may be evaluated. Solution by. trial might therefore proceed just as weli from either of the other two functional relationships contained in the writer'-s diagram.

The writer commends the author's presentation in graph forro oí the values of absolute roughness given iíl the writer's paper, but notes with interest that this plot is consistent with the writer's rather than the author's .choice of basic parameters. Such a graph would therefore have its greatest .value when prepared as

a margimiJ eXtension of the writer's resistance chart, for then no relative-roughness computations would have to be made.

So far as thé author's discussion of open-channel resistance is concerned, the writer takes exception to two points of fundamental importance: First, the author states that such relationships as the Manning formula should'be used in open-channel computations in preference to values derived from pipe tests, implying that the familiar empirical open-channel formulas are inhereiltly more valid. It is known, however, that the .Manning formula (not to mention those of Bazin and Kutter) is not in accordance with the logarithnric -law of relative. roughness upon which tlie author's paper is based. So far as the writer can ascertain, the only reason pipe tests could not generally be used in evaluating open-channel resistance lies irl the fact that few open-channel boundary surfaces are suitable to testing in pipe fÓrm. Aside from the moot question of the effect of cross-sectional shape . (which the empirical open-channel formulas in no way answer), it would appear to the writer that a general resistance graph for uniform open channels should differ little from that for pipes, except in that the familiar parameters *C* and *S* might convimiently be included in the co-ordplate scales; this has been done in'the present form of the writer's chart.

The writer's second objection to the author's closing section is .in regard to his implication that the Froude number should replace the Reynolds number as the fundamental resistance parameter for open-channel flow. So far as boundary resistance is concerne!f, the writer can see no possibility of the Froude number playing a comparable role. It is true that viscous shear is of little significance in comparison with boundary roughness in most open-channel problema, but it is also true that the effect of surface waves upon the interna! resistance to flow has not yet been ascertained. The open-chaim.el problem is, in fact, quite analogous to that of ship resistance, in which the matter of surface drag is considerad wholly independent of the Fraude number. If, to be sure, the phenomena of slug flow, atmospheric drág, and air entrainment prove to govern the resistance in the comparatively infrequent case of supercritical flow in open channels, then the Froude number may well become an appropriate ·resistance criterion, as it already is for cases of channel transition. But to impiy that it should replace the Réynolds number as a resistance parameter whenever á free surface exista seems rather untimely to the writer, in that it could lead to serious misinterpretation of those few principies of boundary resistance which have been definitely established.

P. H. 8CIIWEITZER.[18] . Lest the author's charts, presentad in delightfully handy forma, be used indiscriminately, it is perhaps in arder to add one note of caution. Most of the statements, formulas, and charts are valid only for "long" pipes. For short pipes, the rules controlling turbulence are different, and Reynolds number is not the sole or deciding criterion for the staté of flow.

If the velocity of flow in a long tube is. dec]."eased below the "critica!" value, a change from turbulent to laminar flow takes place rather abruptly. The author sets the indeterminate region between 2000 and 4000 Reynolds number. Even that representa a rather narrow strip in the total range covered by the ·flow of such liquida as water or light oil. Outside of this indeterminate region, the flow is either completely laminar or decidedly turbulent, ignoring the rather thin laminar-boundary layer.

While this is true of relatively long tubes, for short tubes or nozzles it is not. ln a short tube, as was shown by the writer,[19] the normal state may be described as "semiturbulent flow," which may be visualized as a turbulent core in the center and a laminar envelope near the periphery. The thickness of the laminar envelope may vary between wide limita. The change from turbulent tb laminar flow or the reverse takes place in a· short tube so gradually that the intermediate stage usually covers the whole practica! region.

Of course, in both long and short tubes, turbulent flow is promoted by high flow velocity, large tube size, curvatúre of the tube, divergence of the tube, rapid changas in direction and cross-sectional area of the tube. Laminar flow is promoted by hlgh liquid viscosity, la.miilar approacli, rounded entrance to the tube, slight convergence of the tube, absence of curvature and disturbances.

Irrespective of the length of th tube an originally turbulent flow'will remain turbulent, if its Reynolds number R = *vdf.,;* is greater than the critica! Reynolds number; Conversely, an originally laminar flow wil1 remain laminar if R is lower than the · number. ·

If the flow at the entrance is turbulent but its Reynolds Jiumber in the tube is lower than the critica!, the flow wil1 ttl.rn purely laminar if the tube is straight, reasonably smooth, and sufficiently long. If the flow at the entry of the tube is laminar but its Reynolds number is above the critica!, it is hard to predict the character of the ensuing flow. If the entry is smooth and rounded and the tube free _from disturbances and irregularities, the flow will remain laminar even at Reynolds numbers[20] as high as 15,000. ·

In a complete absence of all disturbance's, a laminar flow proba- · ply never turns turbulent, no matter how .high its Reynolds number, but the slightest disturbance will ultimately caq.se tnrbulence if the Reynolds number is above the critica!. The higher the Reynolds number the greater the disturbancé, the shorter the tube travel necessary for turbulence to set in.

In a short tube the critica! Reynolds number is not the one above which the flow generally or in a particular case is turbulent. The flow is frequently laminar at Reynolds numbers above the critica! and it may be turbulent or semiturbulent at Reynolds number below the critica!.

The critica! Reynolds number is the one below which, in a straight long cylindrical tube,. disturbances in the flow will damp out. Above the critica! Reynolds number ·disturbances (approach, entry, etc.) never damp out, no matter how long the tube is. . The critica! ReynolclS number so defined was found by S hiller[21] to be approximátely 2320.

In short tubes, or nozzles, the length is not nearly enough for the flow to assume a stable condition. Under the circumstances, a Reynolds number higher than critica! will have !dendency toward turbulence and vice versa, but it may take a tube travel of 60 times diameter before a atable velocity distribution is developed. The actual flow in the nozzle wil1 be influenced considerably by the state of flow befare the orífice and the disturbances in the approach and within the :r¡¡ozzle. The e1ombination of these factors in addition to the Re:Ynolds number will de ermine the state of turbul nce at the exit of the short tube. For a given short tube or nozzle; the influence of the nozzle factors can be considerad the same; therefore the Reynolds riumber alone will determine the character qf the flow.

With decreasing Reynolds number, the thickness of the laminar layer increases and the turbulent inner portian decreases until it :finally disappears. Itis peculiar to nozzles or short tubes that the change from turbulent to laminar flow (or vice versa)

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19 "Mechanism of Disintegration oí Liquid Jets," by P. H. Schweitzer, *Journal of .A.ppliedPhysics,* vol. 8, 1937, pp. 513-521.

20 With a convergent tube of 10-deg cona angle Gibson (Proceedings of the Royal Society of London; vol. A83, 1910, p. 376), observad, laminar flow at R = 97,000.

21 "Untersuchungen über laminare und turbulenta Stromung," by L. Schiller, *V.DI.Forschunusarbeiten,* vol. 248, 1922.

takes place gradually rather than abruptly. The semiturbulent state extends over a wide range of Reynolds numbers, differing only inthe relative thickness of the turbulent core and laminar .envelope.

. AUTHOR 's 0LOSURE

··The paper was intended for application to normal conditions of engineering practice and specifies a number of qualifications limiting the scope of the charts, S1,1Ch as their restriction to round (straight) new and clean pipes, running full, and with steady fl.ow. " un:der such conditions it was stated, as noted by Professor Par: doe, that the friction factor f '•iS a dimensionless quantity, and at ordinary velocities is a function of two, and only two, other dimensionless quantii'ies,-the relative roughness of the surface and the Reynolds number."

Under abnormal conditions f could of course be affected by other dimensionless criteria. In closed conduits at very high velocities or with rapidly varying pressures it depends .on the Mach or Cauchy number introducing the acoustic velocity. In open channels, as pointed out, free surface phenomena, gravity ·waves, make it logically dependent on Froude's number. At very.low velocities in shallow open troughs it would conceivably be controlled also by the Weber number for surface tension and capillary waves. Capillary forces while important to insects, as to a fl.y on fl.ypaper, are negligible to us in problems of engineering magnitude. Under usual conditions of pipe fl.ow only the two dimensionless ratiosmentioned need be considerad, and it is possible to present the relations between the factors in a chart such as Fig. I.

The discussions have brought out a nuinber of other departures from normal conditions and further limitations to the scope of the charts. Professor Pardoe reminds us that a considerable temperatura difference between the fluid and pipe wall may have a measurable effect on the shear stresses, due to ambiei:tt currents which would increase the momentum transfer in similar manner to turbulent mixing. · , This effect would probably be of importance only at ,the lower Reynolds numbers and with material temperatura differences.

Mr. Pigott reminds us thlit the scope ¡s limited to simple fl.uids and does not cover "queer materials like greases, muds, cement slilrries" and mixtures with suspended solids. Professor (now Commander) Hubbard and Professor Pardoe mention some unusual fonns of pipe surfaces. The author thinks that most of these, including paint coatings, will follow the lines of the charts closely enough for practica! purposes if the proper roughness figures are determined; but the rubber dock-loading hose with helical interna! band will probably follow a curve similar to curve V in Fig. 6, which Colebrook and White obtained for spiral-riveted pipe.

Dr. lppen mentions the rate of increase of roughness from corrosion and gives sorne useful test information. Colebrook found that corrosion usually increases the value of e at substantially a uniform rate with respect to time. Professor Schweitzer calls attentiÓn to the point that the pipe must be long, with an established regime of fiow, and that the charts do not apply to the entrance or "snioothing séction" which require separate allowances. Fortunately we are seldom concerned with close estimates offriction loss in short tubes, where friction is a minor element in the totall ss of head. .

Dr. Ippen's discussion admirably summarizes the basic structure of the charts and gives supporting evidence. His own studies of the problem had, the author believes, led him independently to conclusions similar to Colebrook's.

The Colebrook function has given us a practically satisfactory formulation bridg;ing the previous gap in our. theoretic structure, a region in which the majority of engineering problems fall. It has the further useful property of c·overing in a single formula the whole field of pipe fl.ow above the laminar and critica! zones;· and throughout the field agrees with obse!Vations aselosely as can be reasonably demanded within the range of accuracy available in the measurements, particularly in the evaluation of the boundary roughness.

Referrin:g to a question raised by Professor Daugherty, the inconsistency between Nikuradse's tests in the transition zone and those from commercial pipe is usually attributed either to the close spacing of the artificially applied sand grains, such that one particle may lie in the wake behind another, orto the uniformity of Nikuradsé's particles in contrast. to the usual commercial surface, which is probably a mixture of large and small roughnesses distributed at random The latter explanation seems particularly plausibl , since a few large protuberances mixed with smaller ones could projéct far enough into the laminar boundary !ayer to break it up, while a uniform layer of projections of average size would all rElmain well Vljthin the same thickness of !ayer. Thus Nikuradse's curve clings closely to the smooth pipe line much farther thari the curve for commercial surfaces. At any rate the artificial character of Nikuradse's surfaces weighs agairist the use 9f his values in the region where the discrepancias appear.

Mr. Pigott reviews the progress in charting friction factors-and gives evidence supporting the laws adopted. At the end of his discussion he brings up an interesting question, the form .and location of the dashed line in Fig. 1marking the boundary of the ' rough pipe zone for complete turbulence, beyond which t:he friction factors become practically horizontal. With his gift for detecting relationships he arrives ata modified equation for this curve.

Referring to Figs. 5 and 6 it will be noted that Nikuradse's experiments on artificial roughness gave a curve·which dropped below the "róugh pipe" line andthen approached it from below, while ordinary commercial pipes give points which approach the rough pipe line from above, and that both sets of points seein to merge with the.rough pipe line at about $R\sqrt{f} = 400$, which Rouse accordingly adopted as the equation of the boundary of the rough pipe zone, the dashed line shown in Fig. I. If, however, we adopt the Colebrook furÍction for the transition region to the left of this boundary curve, strictly speaking the Colebrook curves never completely merge with the constant f lines but are asymptotic to them; so that on the basis of the Colej:Jrook function there is no definite boundary to the rough pipe region.

Ptactically however the Colebrook function converges so rapidly to the horizontal! lines that beyond Rouse's dashed curve the differences are insignificant. Considering the practica! difficulties of measurement and consequent scatter of the test points, and the fact that the Colebrook function is partly empirical and merely a satisfactory approximation, it seems hardly justifiable to draw fine distinctions from an extrapolation of this function. If the :fu:D.ction colild be accepted as completely rational it would be more logical to locate the boundary curve so that it would correspond to 'some fixed percentage of excess in f over the f for complete turbulence.

Prompted by Mr. Pigott's suggestion, the author has analyzed the Colebrook equation from this point of view. Calling f the value of the friction factor according to Colebrook, and f_k the value for complete turbúlence according to von Kármán, the Colebrook equation can be expressed

$$1/\sqrt{f_k} - 1/\sqrt{f} = 2 \log \left(1 + \frac{3.7 \times 2.51}{e/D\ R\sqrt{f}}\right).$$

Calling $\frac{3.7}{e/D} \times \frac{2.51}{R\sqrt{f}} = x$ a small quimtity comparad to 1 (of the arder of 0.05 or less in the region of the boundary curve) then log $(1 + x)$ can be expaitded in a series giving

$$0.4343\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots\right)$$ neglecting x^2 and higher powers and

$$\lim_{x \to 0} \frac{\log(1+x)}{x} = 0.8686 \ldots$$

If now we denote by β the *proportional* change in f, that is $\beta = \frac{f_k - f_i}{f_i}$, β being small comparad to 1, then.

$$\frac{1}{\sqrt{f_k}} - \frac{1}{\sqrt{f_i}} = \frac{1}{\sqrt{f_i}(\sqrt{f_k}/\sqrt{f_i} - 1)}$$
$$= \frac{1}{\sqrt{f_i}(\sqrt{1+\beta} - 1)}$$

which, expanding by the binomial formula, is very nearly

$$\frac{1}{\sqrt{f_i}}(1 + \beta/2 - 1) = \beta/2\sqrt{f_i}$$

Hence

$$\beta/2\sqrt{f_i} = 0.8686x = \frac{0.8686 \times 3.7 \times 2.51}{e/D\ R\sqrt{f_i}} - 16.1$$

and $\beta = e/DR$

ia the proportional change inf'causad by the Colebrook function.

Inplotting Fig. 1, the author, instead of continuing indefinitely with the insignificant effect of the small term, and favor.illg the view that f should become substantially constant .in the rough, pipe zone, adoptad the compromise ofignorll¡.g the variation when it fell below about one half of one per cent ¡ and beyond this point the f linea were drawn horizontally at the Kármán value. That is; the chart applied the. Colebrook formula only to the transition zone.

Piltting $\beta = .005 = \frac{e/D}{R}$ we haye $R = \frac{e/D}{.005}$, practieally confirming Mr. Pigott's deduction. If we adopta one per cent variation *off* as a reasonable allowance, the boundary curve could be plotted from $R =$. It might be more logical, to be consistent' with the Colebrook function, to use tliis formula for the bound¡¡.ry curve instead of Rouse's forro. The two curves differ but little, ánd the choice seems more_ a matter of academic preference than practica! importance¡ the scatterof test observations obscuras a final anáwer.

As noted in ni.unerous references m the paper the author has been indebted to Professor Rouse for his contributions to the subject, particulii.i-Iy his valuable paper t th\) I9wa Hydraulic Con-, ference. Professor Rcilliie's inclusion in hiS diacussion of his cha:rt, Fig. 7, from the_ latter papel'; is a usefllll addition to the material here collected. The co-ordinates selected for this chart brlng out the functio l relatimiships in a simple maímer¡ and those who prefer to adopt this forro of chart now have it at hand.

The author still. considera it leas convenient for usual engineering problema than his Fig. 1. While the horizontal scale of Fig. 7 can be expressed in terina of the frictionalloss of head in place of J, this is of no help where the velocity is given and the friction loss is to be found, nor is it of much help in usual engine ring problema where the total head is given and the velocity is to be found. The total head almost always includes not only the friction loss in a pipe system; but also the exit losa, and the loases at entrance and in :fittings, bends, and changas in section¡ and we can seldom assign in advance the value of the friction head or slope of the hydraulic gradient¡ so that successive approximations or trial and-error solutions are still required. While Rouse's chart is. easier to construct, for the reasons explained the author adoptad the forro of Fig. 1as easier to use.

Regarding the author's suggestions as to open··channels, the· questions raiaed by Professor Rouse are probably due mainly to the omission of fuller e'xplanation in the paper. It was not the intention to imply that at low velocities in relatively smooth open channels the friction loss would be independent of Reynolds ilumber, and it may well be found that in this region the logarithmic laws may continÚe to apply, at least in modi:fied forro, and that, as Professor Rouse states, "a general resistance graph for uniform open channels should differ little from that for pipes."

The author was speaking of another region, "open channels dealt with in engineering practica ...usually rough-surfaced and of larga cross section, córresponding to large Reynolds ni.unbers and falling in the zone of complete turbulence." With fairly high velocities, corresponding to large Reynolds numbers, in the presence of a free surface, dimensional considerations require us to include the Fraude number as a criterion¡ and iri the region of complete turbulence we can fortunately afford to omit the Reynolds number as a controlling factor so that we do not have too many variables to handle. The author did not intend to imply that the Fraude number "should replace the Reynolds number as a resistanceJactor *whenever a free surface exists"* but only in the region described, hich however is within the range of ordinary practica.

Professor Rouse recogclzes that free-surface phenomena coroprisa a factor in the problem; his objection to :b:icludirig the Frol1de number is merely that "the effect of surface waves upon the interna! resistance to fl.ow has not yet been ascertained-". which calls on us to investigate the effect rather than to ignore it. Certalnly wave-making resistance is a very real factor both in ship resistance, and in open chanriel fl.ow in the region of the gravitational critica! velocity. Even in tranquil .flow it. still may have. a measurable effect; the location of the maximum velocity point below rather ·than at the surface suggests ·an influence of thia factor.

The !l-Uthor' is con:fident that ProfessorRolise will a: ee with his belief tha further research on open channel friction is much needed¡ and he commends such a project particularly to the civil engineers. Neither the f versus R.charts nor such_formulas as Manning'!3, Kutter's or Bazin's are believed to take. into accoUnt all ofthe m jor controlling factors, and a statiatical analy- of available data alongthe linea suggested, supplemented by· further experimenta, may yield working ·chartei or formulas of great val11e to engineers.

It is regretted that Professor (now Major) Colebrook, who has been serving in the British Army since 1939,.was llDlÍble to submit a discussion. The author wishes to tha:nk all of the discussers fortheir useful contributions, and also to thank Mr. Richard B. Willr'for his able present tion of the paper at the Pittsburgh meeting on behalf of the author.